IOWA End-of-Course Assessment Programs Released Items

ALGEBRA

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- **1** Sally works as a car salesperson and earns a monthly salary of \$2,000. She also earns \$500 for each car (C) she sells. Which equation represents her total monthly income (I) in dollars?
 - **A** I = 12(2,000 + 500C)

INCORRECT: This is an equation that would represent her yearly income in dollars if she sold *C* cars each month.

B I = 2,000 + 500C

CORRECT: The equation that represents Sally's total monthly income is the sum of her monthly salary and the amount she earns for selling *C* cars each month. The 2,000 represents Sally's monthly salary in dollars and 500C represents the amount in dollars Sally earns for selling *C* cars each month: I = 2,000 + 500C

C I = (2,000 + 500)C

INCORRECT: This is an equation that would represent Sally earning \$2,500 (simplified \$2,000 + \$500) per car she sold. It does not take into account a monthly salary.

D I = 2,000C + 500

INCORRECT: This is an equation that would represent Sally having a monthly salary of \$500 and earning \$2,000 per car she sold.

CCSS Conceptual Category: Algebra

CCSS Domain:

2 What is the solution to $2(3 - x) \le 8$?

A $x \leq -1$

INCORRECT: The student did not flip the direction the inequality sign points when dividing by a negative number within an inequality.

B $x \ge -1$

CORRECT: One possible explanation. Problem may also be solved by dividing both sides by 2 first and then solving for *x*. $2(3 - x) \le 8$

$6 - 2x \le 8$	Distribute the 2.
$-2x \le 2$	Subtract 6 from both
	sides.
$x \ge -1$	Divide both sides by -2 .
	D 11

Recall when dividing
by a negative number
within an inequality, the
direction the inequality
sign points is flipped

C $x \le 1$

INCORRECT: The student divided incorrectly and therefore did not flip the direction the inequality sign points.

D $x \ge 1$

INCORRECT: The student divided incorrectly.

CCSS Conceptual Category:

Algebra

CCSS Domain:

3 What is the solution set to the following equation?

 $x^2 + 2x - 15 = 0$

A {-15, 2}

INCORRECT: The student used the coefficient of the linear term and the constant from the equation as the elements of the solution set.

B {-10, 6}

INCORRECT: The student did not divide the simplified numerator in the quadratic formula by 2 (2 \times 1).

C {-5, 3}

CORRECT: One possible explanation using factoring. Problem may also be solved with quadratic formula, completing the square, or other methods.

 $x^{2} + 2x - 15 = 0$ (x + 5)(x - 3) = 0 x + 5 = 0 or x - 3 = 0 x = {-5, 3}

D {-3, 5}

INCORRECT: The student used the constant terms of the binomials (using the factoring method) as the elements of the solution set.

CCSS Conceptual Category:

Algebra

CCSS Domain:

4 What is the solution to $\sqrt{x} - 2 = 4$?

A 6

INCORRECT: The student only added 2 to both sides of the equation.

B 12

INCORRECT: The student multiplied 6 by 2 instead of squaring.

\mathbf{C} $\overline{20}$

INCORRECT: The student incorrectly squared each term first and then solved x - 4 = 16.

D 36

CORRECT: $\sqrt{x} - 2 = 4$ $\sqrt{x} = 6$ Add 2 to both sides. x = 36 Square both sides.

CCSS Conceptual Category:

Algebra

CCSS Domain:

5 Maria is having a birthday party for her friend, Miguel. A total of 50 people will be at the party. She currently has eight sodas. Which inequality can be used to determine how many 12-packs of soda (x) she should buy so that each person can have at least two sodas?

A $x + 8 \ge 50$

INCORRECT: The student did not multiply the number of people by 2 to determine the least number of sodas she needs and did not multiply the number of 12-packs by 12 to represent the total number of sodas she buys.

B $12x + 8 \ge 50$

INCORRECT: The student did not multiply the number of people by 2 to determine the least number of sodas she needs.

C $x + 8 \ge 100$

INCORRECT: The student did not multiply the number of 12-packs by 12 to represent the total number of sodas she buys.

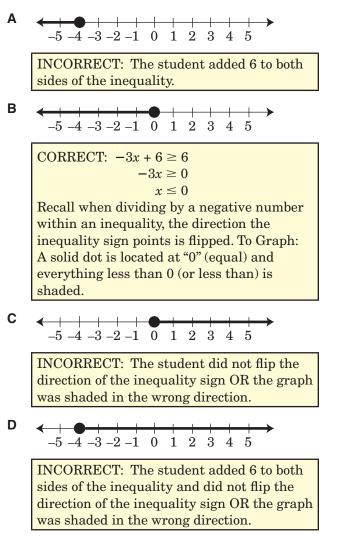
D $12x + 8 \ge 100$

CORRECT: The inequality that represents the situation compares the number of sodas; the number she should buy and the number she needs. There are going to be a total of 50 people at the birthday party and Maria wants enough sodas so each person can have at least 2, so the number that she should buy needs to be greater than or equal to 100. Since *x* represents the number of 12-packs of soda then 12x represents the total number of sodas she buys from the 12-packs. That number of sodas in addition to the 8 she already has needs to be at least 100: $12x + 8 \ge 100$.

CCSS Conceptual Category: Algebra

CCSS Domain:

6 Which graph represents the solution to $-3x + 6 \ge 6$?



CCSS Conceptual Category:

Algebra

CCSS Domain:

- 7 When solving $x^2 = 24 10x$ by factoring, what pair of equations will be solved in the final step?
 - **A** x 4 = 0x - 6 = 0

INCORRECT: The student did not change the linear and constant terms to their opposites when setting the equation equal to 0: $x^2 - 10x + 24 = 0.$

- **B** x 2 = 0
 - x + 12 = 0

CORRECT: $x^2 = 24 - 10x$ $x^2 + 10x - 24 = 0$ Set equation equal to 0. (x - 2)(x + 12) = 0 Factor. x - 2 = 0 or x + 12 = 0 Set factors equal to 0.

- **C** x + 2 = 0
 - x 12 = 0

INCORRECT: The student set the equation equal to 0 correctly, but factored signs incorrectly OR did not change the linear term to its opposite when setting the equation equal to $0: x^2 - 10x - 24 = 0$.

D x + 4 = 0

x + 6 = 0

INCORRECT: The student did not change the constant term to its opposite when setting the equation equal to 0: $x^2 + 10x + 24 = 0.$

CCSS Conceptual Category: Algebra

CCSS Domain:

8 If x = -3, what is the value of $x^2 + 3(x + 2)$?

A -16

INCORRECT: The student squared the 3 and then made it its opposite and distributed the 3 incorrectly to only the first term in the quantity: $(-3)^2 + 3(-3 + 2) \rightarrow -9 + -9 + 2.$

B -12

INCORRECT: The student squared the 3 and then made it its opposite: $(-3)^2 + 3(-3 + 2) \rightarrow -9 + 3(-1).$

С

 $\mathbf{2}$

INCORRECT: The student distributed the 3 incorrectly to only the first term in the quantity: $(-3)^2 + 3(-3 + 2) \rightarrow 9 + -9 + 2$.

D

CORRECT: $x^2 + 3(x + 2) = (-3)^2 + 3(-3 + 2)$ = 9 + 3(-1) = 9 - 3 = 6

CCSS Conceptual Category:

Algebra

CCSS Domain:

Arithmetic with Polynomials and Rational Expressions

9 Sam purchased two bottles of water and three hot dogs at the ballpark for \$8.50. Mary purchased one bottle of water and two hot dogs for \$5.25. What system of equations could be solved to determine the prices in dollars of a hot dog (*h*) and a bottle of water (*w*)?

A $\begin{cases} 2w + 3h = 8.50 \\ w + 2h = 5.25 \end{cases}$

CORRECT: The first equation needs to represent the cost in dollars of 2 bottles of water and 3 hot dogs for a total of 8.50 where w is the price per bottle of water and h is the price per hot dog: 2w + 3h = 8.50. The second equation needs to represent the cost in dollars of 1 bottle of water and 2 hot dogs for a total of 5.25: w + 2h = 5.25.

B $\begin{cases} 3w + 2h = 8.50 \\ w + 2h = 5.25 \end{cases}$

INCORRECT: The student reversed the variables in the first equation (represented the cost in dollars for 3 bottles of water and 2 hot dogs).

C $\begin{cases} w + h = 8.50 \\ w + h = 5.25 \end{cases}$

INCORRECT: The student did not take into account the number of bottles of water or hot dogs in either equation.

D $\begin{cases} 3w + 2h = 8.50 \\ 2w + h = 5.25 \end{cases}$

INCORRECT: The student reversed the variables in both equations (represented the cost in dollars for 3 bottles of water and 2 hot dogs and 2 bottles of water and 1 hot dog).

CCSS Conceptual Category: Algebra

CCSS Domain:

- **10** The equation y = -5.25x + 320.75 models the amount of money (y) in dollars Jillian has at the end of x weeks. Which scenario is modeled by the equation?
 - **A** Jillian owes \$320.75 and is paying \$5.25 each week.

INCORRECT: The student chose the scenario that would be modeled by the equation y = -5.25x - 320.75.

B Jillian has \$320.75 and is paying \$5.25 each week.

CORRECT: The *y*-intercept of the equation represents the amount of money in dollars Jillian starts with. Since this is a positive value, she has \$320.75. The slope of the equation represents the change in the amount of money she has each week. Since this is a negative value, she is paying \$5.25 each week.

C Jillian owes \$320.75 and is receiving \$5.25 each week.

INCORRECT: The student chose the scenario that would be modeled by the equation y = 5.25x - 320.75.

D Jillian has \$320.75 and is receiving \$5.25 each week.

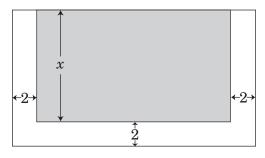
INCORRECT: The student chose the scenario that would be modeled by the equation y = 5.25x + 320.75.

CCSS Conceptual Category:

Algebra

CCSS Domain:

11 The diagram shows a rectangular garden with a two-foot-wide sidewalk on three sides. The length of the garden is three times its width. If the garden's width in feet is x, what expression represents the area in square feet of the sidewalk?



A 8x + 4

INCORRECT: The student incorrectly represented the length of the rectangle formed by the garden and the sidewalk as (3x + 2).

B 10x + 8

CORRECT: The area of a rectangle is determined by multiplying the length and the width of the rectangle. The width of the rectangular garden is *x* and the length is 3x, so the area of the garden is represented by $3x \cdot x = 3x^2$. The width of the rectangle formed by the garden and the sidewalk is (x + 2) and the length of the rectangle formed by the garden and the sidewalk is (3x + 4), so the area of the rectangle formed by the garden and the sidewalk is represented by $(x + 2)(3x + 4) = 3x^2 + 10x + 8$. Subtracting the area of just the garden from the expression representing the rectangle formed by the garden and the sidewalk results in the expression for the area of only the sidewalk: $3x^2 + 10x + 8 - 3x^2 = 10x + 8$.

C 14x + 8

INCORRECT: The student incorrectly represented the width of the rectangle formed by the garden and the sidewalk as (x + 4) and the length as (3x + 2).

D 16x + 16

INCORRECT: The student incorrectly represented the width of the rectangle formed by the garden and the sidewalk as (x + 4).

CCSS Conceptual Category:

Algebra

CCSS Domain:

12 Emily's family needs to rent a moving truck to move their belongings to a different house. The rental cost for Truck A is \$42 per day plus \$.69 per mile and the rental cost for Truck B is \$30 per day plus \$.79 per mile. If Emily's family rents the moving truck for two days, how many miles would they need to drive so that the rental cost for the two trucks would be the same?

A 60

INCORRECT: The student took 2 times the rates per mile and solved: 42 + 1.38x = 30 + 1.58x

B 120

INCORRECT: The student solved the problem if the trucks were rented for only 1 day and not 2:42 + .69x = 30 + .79x.

C 180

INCORRECT: The student incorrectly subtracted 42 and $30: 42 - 30 \rightarrow 18$.

D 240

CORRECT: For two days, the cost in dollars (y) to rent Truck A is represented by the equation y = 84 + .69x where x is the number of miles and the cost in dollars to rent Truck B is represented by the equation y = 60 + .79x. To determine how many miles Emily's family needs to drive for the rental costs to be the same, set the equations equal to each other and solve for x. 84 + .69x = 60 + .79x24 = .10xx = 240 miles

CCSS Conceptual Category: Algebra

CCSS Domain:

- 13 What is the equation of a line that is parallel to -x + 2y = 8 and has the same y-intercept as 3x + 2y = -6?
 - **A** y = -2x 3

INCORRECT: The student found the slope of the line perpendicular to the line represented by -x + 2y = 8.

B y = -x - 6

INCORRECT: The student used the coefficient for x from the first equation as the slope and the constant from the second equation as the *y*-intercept of the line.

C
$$y = \frac{1}{2}x - 3$$

CORRECT: Since the line is parallel to the line represented by -x + 2y = 8 the lines have the same slope. The slope of the line represented by -x + 2y = 8 is $\frac{1}{2}(y = \frac{1}{2}x + 4$ in slope-intercept form). The *y*-intercept of 3x + 2y = -6 is $-3(y = -\frac{3}{2}x - 3$ in slope-intercept form). Therefore the equation of the line is $y = \frac{1}{2}x - 3$.

D $y = \frac{1}{2}x - 6$

INCORRECT: The student incorrectly found the *y*-intercept of 3x + 2y = -6 to be -6 (did not divide by the coefficient of the *y*-term).

CCSS Conceptual Category:

Functions

CCSS Domain:

Building Functions

14 Solve
$$2x^2 + 5x - 1 = 0$$
.

A
$$x = \frac{-5 \pm \sqrt{17}}{2}$$

INCORRECT: The student did not multiply by 2 in the denominator and simplified incorrectly inside the square root; simplified as 25-8.

$$\mathbf{B} \quad x = \frac{-5 \pm \sqrt{33}}{2}$$

INCORRECT: The student did not multiply by 2 in the denominator.

C
$$x = \frac{-5 \pm \sqrt{17}}{4}$$

INCORRECT: The student simplified incorrectly inside the square root; simplified as 25-8.

D
$$x = \frac{-5 \pm \sqrt{33}}{4}$$

CORRECT: Substituted into and simplified the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. $x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-1)}}{2(2)}$ $x = \frac{-5 \pm \sqrt{25 + 8}}{4}$ $x = \frac{-5 \pm \sqrt{33}}{4}$

CCSS Conceptual Category:

Algebra

CCSS Domain:

15 What value of *y* makes the following system of equations true?

$$\begin{cases} 3x + 6y = 12 \\ 5x - 6y = 4 \end{cases}$$

A 0

INCORRECT: The student thought since the *y*-term was eliminated when adding the equations, y = 0.

В

1

CORRECT: Explanation provided using the addition method. Other methods are possible.

3x + 6y = 12+ 5x - 6y = 48x + 0y = 168x = 16x = 2

Substituting x = 2 into either one of the original equations and solving for *y*.

C 2

INCORRECT: The student found the value of *x*.

D 3

INCORRECT: The student incorrectly solved the first equation for *y*; added 12 and 6 then divided by 6.

CCSS Conceptual Category:

Algebra

CCSS Domain: